Sample Space - Sample space is a set of all outcomes and is represented by the symbol S. in a sample space an outcome in a sample space is called an element or sample point. To note sample space has a finite number of elements, below is how we can show a sample space when a coin is flipped.  
S = {H, T}

For tossing a die the sample space is S1 = {1,2,3,4,5,6,} now if we must find whether the number is odd or even the sample space S2 = {even, odd} We can show a list of sample spaces in a systematic manner in terms of tree diagrams.

Event -We are interested in finding an occurrence of certain event in a sample space. An event is a subset of a sample space. For instance, we may be interested in the event A that when a die is rolled the outcome is divisible by 3. S = {3,6}.

The intersection of two events A and B is denoted by A intersection B  it contains all elements that are common to both A and B.

If two events are mutually exclusive or disjoint  - means A and B have no elements in common.

The union of two events, A and B, is denoted by  - A or B or both.

For e.g. A = {a, b, c} and B = {b, c, d,e} A U B = { a,b,c,d,e}

Several results are as follow:  
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Fig 1 – some operations

Counting Sample Points -To calculate, if an operation O1 can be perform in n1 ways and O2 can be performed in n2 ways then both O1 and O2 can be performed together in n1\*n2 ways.

Ex Number of sample points pair of dice is thrown together is 6\*6 = 36 ways

Ex If a 22-member club needs to elect a chair and treasurer, how many ways can two be selected?

N1 = 22 n2 = 22-1 = 21 therefore there are 462 diverse ways this can be performed.

This multiplication rule can be extended for different operations O1, O2, O3, O4 …. Ok the sequence of k operations can be done in n1\*n2\*n3\*n4…. \*Nk ways.

A permutation is an arrangement for all or a part of set of objects  
  
Factorial of n – n! = n\* (n-1) \*(n-2) \*…. \*2\*1

Also note 0! =1

N! is the number of permutations of n objects.

When number of permutations of n distinct objects taken r at a time is nPr = n! /(n-r)!

Example – How many selections are to select 3 students out of 25 for award?

nPr = 25P3 = 25! /22! = 13800

For arranging objects, the number of permutations (n-1)!

To find number of distinct permutations to arrange k things out of n distinct objects. Permutation is used where order is important

A line with letters and numbers

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Fig 2: Permutation

The number of combinations of distinct objects taken r at a time is

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Fig 3: Combination

Combination is used where order isn’t important

How many different letter arrangements can be made from letter STATISTICS?

10! / 3! 3! 2! 1! 1! = 50400

Probability of an Event

It is basically chance of occurrence of events denoted by P   
p = n(a)/ n(S) = Favorable event / Sample space

Example probability of getting a head when a coin is tossed

S= {h,t}

P(h) = ½

P(t) = ½

Also, P(h) + P(t) =1

Axioms of Probability

Axiom 1: Probability lies between 0 and 1 i.e 0<= p<=1 -> It can never be negative or greater than 1

Axiom 2: sum of all probabilities = 1 sigma P =1

Axiom 3: P(A) + P(A’) = 1

P(A’) = compliment of P(A)

If you throw a dice the sample space of the dice is s = (1,2,3,4,5,6)

P (1) = P (2) = P (3) = P (4) = P (5) = P (6) = 1/6 and their summation is 6 \* 1/6 which gives us 1

Anytime we are dealing with probability the summation = 1

Let’s say we have two coins, and we throw them, the sample space will be S= (HH, HT, TH, HH)

Additive Rule in Probability – often we can find the probability of some event from other event for instance for the Venn diagram A diagram of two circles

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P ( A U B ) = P(A) + P(B) – P(A intersection B)

If A and B are mutually exclusive, then P ( A U B ) = P(A) + P(B)

Conditional probability

“ The probability of an event B occurring when it is known that some event A has occurred is called a conditional probability “

Denoted by P (B|A) – prob of B given A

Eg if we need to find P ( C|A) = it means probability of C given A

A close-up of a number

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Fig 4 : Conditional Probability

To prove if two events A and B are independent – we need to show P(B|A)=P(B) or P(A|B)=P(A).

In an experiment where both events A and B can happen, the probability that both A and B occur together, denoted as P(A ∩ B), is found by multiplying the probability of A occurring, P(A), by the probability of B occurring given that A has already happened, P(B | A). This formula holds as long as P(A) is greater than zero.

Bayesian statistics

Bayesian stats is very powerful approach to statistical inference – applies prob theory to analyze experimental data in scientific and engineering fields.

Total Probability theorem

It helps to compute the probability of event A when the sample space is divided into mutually exclusive events b1, b2, b3, b4 up to bk , it states that:

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Fig 5: Total Probability theorm

Bayes Rule

Now Instead of focusing to find P(A), if we need to find (P (Bi|A)) – then there comes bayes rule.  
Central idea around bayes rule- If we know how likely each partition event Bᵢ is and how A behaves under each Bᵢ, we can find the probability of a specific Bᵣ given that A happened.

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Fig 6: Bayes rule

References

R. E. Walpole, R. H. Myers, S. L. Myers, and K. Ye, *Probability & Statistics for Engineers & Scientists*, 9th ed. Prentice Hall